An Introduction to Java Geometry Expert*
(Extended Abstract)

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Abstract. This paper gives a brief introduction to the system Java Geometry Expert (JGEX). This system consists of three parts: the drawing part, the proving and reasoning part, and the most distinctive part – the part for generating *visually dynamic presentation of proofs* in plane geometry. The current version of JGEX is beta 0.80, which is available at our website woody: `http://woody.cs.wichita.edu`.

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1 Introduction

Highly successful algebraic methods for automated geometry theorem proving have been developed since Wu's pioneering work in 1978 [16]. Hundreds of difficult geometry theorems have been proved with these methods [7,9,10,4,15,14].

The proofs, generated by these algebraic methods, involve computations of polynomials with hundreds or even thousands of terms. Thus they are generally not (human) readable.

Most proofs in geometry textbooks are synthetic (possibly with some very simple algebraic computations). Students can read the proofs step by step with assistance of one or more diagrams. However, they often need to spend time and energy on identifying a geometry element in the proof text with that in the corresponding diagram. When the same element is mentioned later in the proof text they might spend equal amounts of time and energy on identifying it again in the diagram. When the diagram becomes complicated, e.g., there are over a dozen of points involved in the diagram, the problem becomes serious not only to novices, but also to experts.

Geometry textbooks generally alleviate this problem by using two or more diagrams with different marks for angles and segments, and possibly with shadowed areas, e.g., a shadowed triangle, in the diagrams. However, this kind of presentation of proofs is static.

With dynamic mediums such as computer displays, we propose an entirely new approach – visually dynamic presentation of proofs (VDPP), to solve this problem.

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In a single diagram for the proof, when the proof text goes on step by step with mouse clicks, the related geometry elements in the diagram are animated, added, or deleted dynamically with various visually dynamic effects.

We have implemented two methods for generating such presentations of proofs in our developing system Java Geometry Expert (JGEX): the manual input method and the automated method, which will be illustrated by examples in later sections.

2 The Parts of JGEX

The Drawing Part [5]. This part has the dynamic geometry features similar to those used in the popular and excellent systems such as the Geometers Sketchpad, Cabri, and Cinderella.

With mouse clicks, the diagram is constructed and the corresponding geometry statement is generated with its non-degenerate conditions in its *geometric form*. It can be saved in several forms. One form is in plain text, which can be used in turn to generate the diagram with prompting the user to select points.

The Proving and Reasoning Part. Beside the traditional algebraic methods such as Wu's method and the Gröbner basis method, we have also implemented the full-angle method and the deductive database method [3,6] for generating short, elegant synthetic proofs. These are the basis of the next part.

The Part of the Visually Dynamic Representation of Proofs. We have implemented two methods for generation of such visually dynamic representation (VDPP) which will be discussed in detail in next two sections.

3 The Manual Input Method

JGEX provides a very *general* tool for manually creating VDPPs. It can be used by students or teachers to write or to present proofs. We plan to implement four modes. So far we have only implemented Modes 1 and 2.

Mode 1: Animated Diagrams Only. The approach in this mode is very similar to the approach of Proof Without Words (PWW) presented in three excellent books [11,12,13]. However, we add another dimension to the PWW approach, i.e., instead of a static diagram or a series of static diagrams, the diagram here is visually dynamic.

Example 1. A Proof of the Pythagorean Theorem (Fig. 1).

This proof is from the webpage [1] which is a collection of 72 proofs of the Pythagoras theorem. From our visually dynamic presentation in Fig. 1, one can clearly see the elegance of the proof. For the real animated gif file created with JGEX see the Collection at `http://woody.cs.wichita.edu/collection`, where many JGEX-manually created examples are given, in particular there are over two dozens of proofs of the Pythagorean Theorem with the area dissection

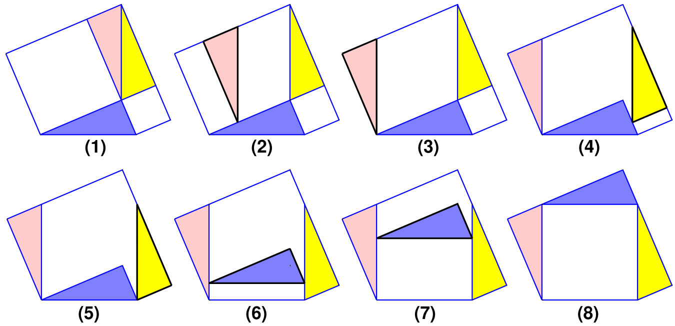

Fig. 1. A Proof of Pythagorean Theorem with Only Three Moves

method, similar to this one, are given. These examples can be easily created with JGEX as a general tool.

Mode 2: Proofs with text and an animation diagram. First a student needs to know the proof. Then he/she inputs the proof step by step mostly with mouse clicks on the diagram to avoid typos (e.g., typing letter A instead of letter S). When the proof is completed, others can see the proof step by step while corresponding geometry elements in the step are animated to reflect the geometric meaning of this step.

In this mode, JGEX only verifies numerical correctness of the assertion of a step by randomly generating many floating point number instances of the diagram. It does not care whether an assertion is a logical consequence of previous assertions. It could be a good tool for students to write proofs. However, the teachers need to decide whether the proof is correct or complete.

Example 2. Let circle O be the circumscribed circle of an equilateral triangle ABC and E a point on the arc AB. Prove that $EC = EA + EB$ (Fig. 2).

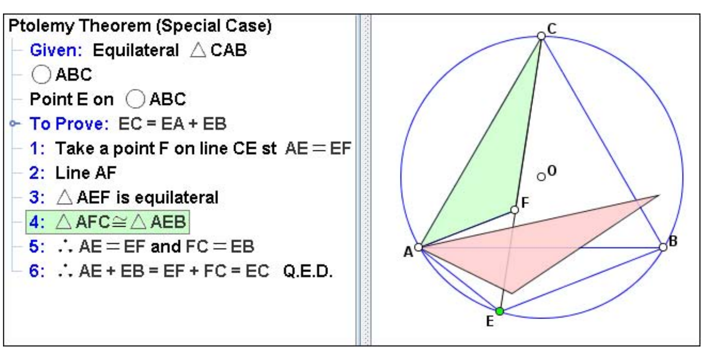

Fig. 2. A Proof of the Special Case of Ptolemy's Theorem

In Fig. 2, the fact $\triangle AFC \cong \triangle AEB$ is highlighted with the visual effects as follows: triangle $\triangle AFC$ and triangle $\triangle AEB$ are filled with colors and a copy of $\triangle AFC$ is rotated on the fly and drops to $\triangle AEB$. Fig. 2 is a static diagram from the animated diagram of this theorem. In this diagram, the red color-filled triangle is rotating and about to drop to the $\triangle AEB$.

4 The Automated Methods

In JGEX, we have implemented automated generation of VDPPs with the full-angle method [3] and with the deductive database method [6]; for details, see our paper [17] which is already published. The two methods developed in the 1990s imply automated addition of auxiliary geometric elements: given two points A and B, three non-collinear points C, D and E, or two lines l_1 and l_2, if the automatically generated proof requires, there will be a line AB or a segment AB, or a circle CDE, or a full-angle $\angle[l_1, l_2]$, etc. However, the methods are unable to add a point of intersection of, say, two given lines.

The full-angle method is a natural way to generate proofs with hierarchical structures. Any non-initial facts found by forward chaining can be expanded to view the proof of the fact for further investigation. If the proof of this fact has a sufficient number of steps, we can consider this fact and its proof as a lemma application. Hierarchically structured proofs allow the user to concentrate on the main steps.

Example 3. (Simson's Theorem) Let E be a point on the circumscribed circle(D) of triangle ABC. Let F, G, and H be the feet of the three perpendicular lines

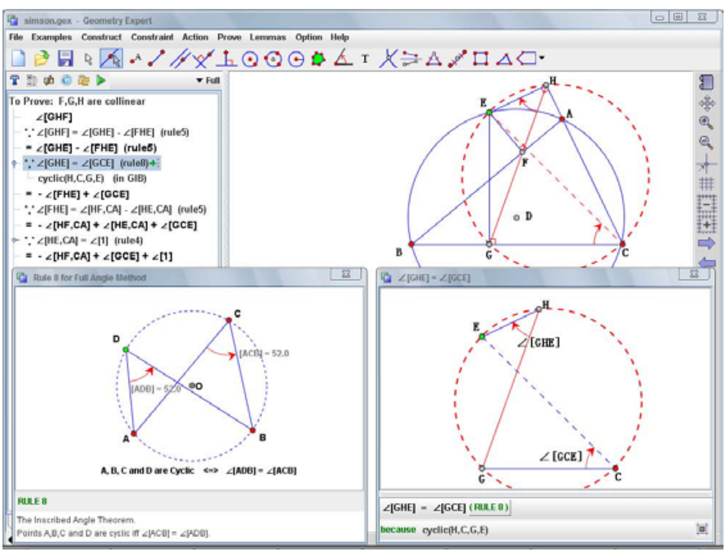

Fig. 3. Simson's Theorem

from point E to the three sides AB, BC, and AC, respectively. Show that F, G and H are collinear (Fig. 3).

Fig. 3 shows the machine-generated proofs with the full-angle method. Step 2 is expanded and highlighted with the two auxiliary angles and one auxiliary circle appear and blink. This step uses Rule 8 (See Fig. 3) with the fact $cyclic(C, E, G, H)$. This fact is found by the forward chaining, i.e., a fact in the fixpoint. The user can expand this step to view the proof of this fact.

There are two floating windows in Fig. 3. The right one shows the portion of the diagram where the rule applies. The left one gives the detail of the rule. In this case, it shows that Rule 8 is the full-angle version of the inscribed angle theorem.

5 Visualization of Fixpoints

The fixpoint generated by the deductive database method contains surprisingly rich amounts of information, some of which is very unexpected. Visualizing fixpoints can help users to explore properties that they are not aware of.

Example 4. (The Orthocenter Theorem) Let CD and BE be two altitudes of triangle ABC, F the intersection of CD and BE, and G the intersection of AF and BC. Show that $\angle[DGA] = \angle[AGE]$ (Fig. 4).

Fig. 4 shows the fixpoint of this theorem found by forward chaining. There are seven groups of angle congruence. By clicking one of them (highlighted in

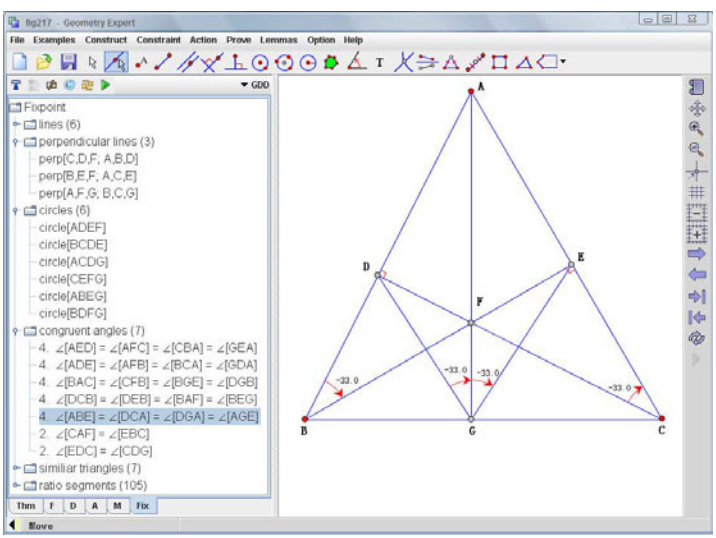

Fig. 4. The Orthocenter Theorem

Fig. 4), the corresponding angles appear in the diagram. We can see that the fact ($\angle[DGA] = \angle[AGE]$) is in the fixpoint thus the conclusion is reached by forward chaining.

6 Conclusion

JGEX is based on our previous version of Geometry Expert (GEX)[8]. However, it has been rewritten completely in Java with emphasis on its ease of use. The most distinctive feature of JGEX is its visually dynamic presentation of proofs. This makes JGEX a valuable tool for generating and presenting geometry proofs with various visual effects. It could have many applications, e.g., in geometry education.

JGEX is still an ongoing developing system. The current version is *b*eta 0.80 which is available in our website woody [2].

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