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>
> read "AddOreSatoImple.mm" :
Warning, (in IntegerLinearType V1) `i` is implicitly declared local
MultivariateLD test:539
Warning, (in IntegerLinearType V2) `i` is implicitly declared local
MultivariateLD test:989
Warning, (in IntegerLinearType) `i` is implicitly declared local
MultivariateLD test:2408
> with(AddOreSato);
[AbramovReduction, CompletePartialFraction, IntegerLinearTypePoly, PartialFraction,
SortFraction, TransUniWZForms, VerifyWZForm, VerifyWZFormDecomp, WZFormDecomp]

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> f := normal( add( 1 / (4x + 6y + 5z + l), l=0..3 ) );
f := (2 (128 x^3 + 576 x^2 y + 480 x^2 z + 864 x y^2 + 1440 x y z + 600 x z^2 + 432 y^3 + 1080 y^2 z
+ 900 y z^2 + 250 z^3 + 144 x^2 + 432 x y + 360 x z + 324 y^2 + 540 y z + 225 z^2 + 44 x + 66 y
+ 55 z + 3)) / ((4 x + 6 y + 5 z) (4 x + 6 y + 5 z + 1) (4 x + 6 y + 5 z + 2) (4 x + 6 y
+ 5 z + 3))

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> g := normal( add( 1 / (4x + 6y + 5z + l), l=0..5 ) + add( 1 / (3y + 2z + l), l=0..2 ) );
g := (110592 x^6 y^2 + 147456 x^6 y z + 49152 x^6 z^2 + 1161216 x^5 y^3 + 2488320 x^5 y^2 z
+ 1769472 x^5 y z^2 + 417792 x^5 z^3 + 4976640 x^4 y^4 + 14722560 x^4 y^3 z + 16277760 x^4 y^2 z^2
+ 7971840 x^4 y z^3 + 1459200 x^4 z^4 + 11197440 x^3 y^5 + 42301440 x^3 y^4 z + 63763200 x^3 y^3 z^2
+ 47934720 x^3 y^2 z^3 + 17971200 x^3 y z^4 + 2688000 x^3 z^5 + 13996800 x^2 y^6 + 64385280 x^2 y^5 z
+ 123171840 x^2 y^4 z^2 + 125426880 x^2 y^3 z^3 + 71701200 x^2 y^2 z^4 + 21816000 x^2 y z^5
+ 2760000 x^2 z^6 + 9237888 x y^7 + 50108544 x y^6 z + 116313408 x y^5 z^2 + 149765760 x y^4 z^3
+ 115522200 x y^3 z^4 + 53379000 x y^2 z^5 + 13680000 x y z^6 + 1500000 x z^7 + 2519424 y^8
+ 15746400 y^7 z + 43005168 y^6 z^2 + 67033008 y^5 z^3 + 65221200 y^4 z^4 + 40560750 y^3 z^5
+ 15744375 y^2 z^6 + 3487500 y z^7 + 337500 z^8 + 73728 x^6 y + 49152 x^6 z + 1244160 x^5 y^2
+ 1769472 x^5 y z + 626688 x^5 z^2 + 7361280 x^4 y^3 + 16277760 x^4 y^2 z + 11957760 x^4 y z^2
+ 2918400 x^4 z^3 + 21150720 x^3 y^4 + 63763200 x^3 y^3 z + 71902080 x^3 y^2 z^2 + 35942400 x^3 y z^3
+ 6720000 x^3 z^4 + 32192640 x^2 y^5 + 123171840 x^2 y^4 z + 188140320 x^2 y^3 z^2
+ 143402400 x^2 y^2 z^3 + 54540000 x^2 y z^4 + 8280000 x^2 z^5 + 25054272 x y^6 + 116313408 x y^5 z
+ 224648640 x y^4 z^2 + 231044400 x y^3 z^3 + 133447500 x y^2 z^4 + 41040000 x y z^5
+ 5250000 x z^6 + 7873200 y^7 + 43005168 y^6 z + 100549512 y^5 z^2 + 130442400 y^4 z^3
+ 101401875 y^3 z^4 + 47233125 y^2 z^5 + 12206250 y z^6 + 1350000 z^7 + 8192 x^6 + 387072 x^5 y
+ 270336 x^5 z + 3732480 x^4 y^2 + 5460480 x^4 y z + 1989120 x^4 z^2 + 14826240 x^3 y^3
+ 33384960 x^3 y^2 z + 24986880 x^3 y z^2 + 6215680 x^3 z^3 + 28771200 x^2 y^4 + 87842880 x^2 y^3 z

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$$\begin{aligned}
& + 100362240 x^2 y^2 z^2 + 50852160 x^2 y z^3 + 9640800 x^2 z^4 + 27200448 x y^5 + 105053760 x y^4 z \\
& + 162032400 x y^3 z^2 + 124748640 x y^2 z^3 + 47939400 x y z^4 + 7356000 x z^5 + 10054368 y^6 \\
& + 47017584 y^5 z + 91494360 y^4 z^2 + 94831020 y^3 z^3 + 55212300 y^2 z^4 + 17120250 y z^5 \\
& + 2208750 z^6 + 30720 x^5 + 737280 x^4 y + 529920 x^4 z + 4708800 x^3 y^2 + 7015680 x^3 y z \\
& + 2603520 x^3 z^2 + 12564720 x^2 y^3 + 28661040 x^2 y^2 z + 21738240 x^2 y z^2 + 5481600 x^2 z^3 \\
& + 15085440 x y^4 + 46510200 x y^3 z + 53675460 x y^2 z^2 + 27478800 x y z^3 + 5265000 x z^4 \\
& + 6750540 y^5 + 26273160 y^4 z + 40844655 y^3 z^2 + 31702725 y^2 z^3 + 12285000 y z^4 \\
& + 1901250 z^5 + 43520 x^4 + 650880 x^3 y + 477440 x^3 z + 2751408 x^2 y^2 + 4156704 x^2 y z \\
& + 1564608 x^2 z^2 + 4507488 x y^3 + 10390680 x y^2 z + 7966296 x y z^2 + 2031056 x z^3 \\
& + 2546964 y^4 + 7918776 y^3 z + 9217611 y^2 z^2 + 4760574 y z^3 + 920370 z^4 + 28800 x^3 \\
& + 273312 x^2 y + 203808 x^2 z + 697680 x y^2 + 1066896 x y z + 406584 x z^2 + 532008 y^3 \\
& + 1238436 y^2 z + 958986 y z^2 + 246990 z^3 + 8768 x^2 + 48096 x y + 36448 x z + 55656 y^2 \\
& + 86232 y z + 33300 z^2 + 960 x + 2160 y + 1680 z) / ((4 x + 6 y + 5 z) (4 x + 6 y + 5 z \\
& + 1) (4 x + 6 y + 5 z + 2) (4 x + 6 y + 5 z + 3) (4 x + 6 y + 5 z + 4) (4 x + 6 y + 5 z \\
& + 5) (3 y + 2 z) (3 y + 2 z + 1) (3 y + 2 z + 2))
\end{aligned}$$

$$\begin{aligned}
& \text{h} := \text{normal}\left(\text{add}\left(\frac{1}{4 x + 6 y + 5 z + l}, l=0..4\right) + \text{add}\left(\frac{1}{3 y + 2 z + l}, l=0..1\right)\right); \\
& \text{h} := (6144 x^5 y + 4096 x^5 z + 57600 x^4 y^2 + 84480 x^4 y z + 30720 x^4 z^2 + 207360 x^3 y^3 \\
& + 472320 x^3 y^2 z + 357120 x^3 y z^2 + 89600 x^3 z^3 + 362880 x^2 y^4 + 1123200 x^2 y^3 z \\
& + 1300320 x^2 y^2 z^2 + 667200 x^2 y z^3 + 128000 x^2 z^4 + 311040 x y^5 + 1218240 x y^4 z \\
& + 1905120 x y^3 z^2 + 1486800 x y^2 z^3 + 579000 x y z^4 + 90000 x z^5 + 104976 y^6 + 497664 y^5 z \\
& + 981720 y^4 z^2 + 1031400 y^3 z^3 + 608625 y^2 z^4 + 191250 y z^5 + 25000 z^6 + 1024 x^5 \\
& + 26880 x^4 y + 19200 x^4 z + 161280 x^3 y^2 + 241920 x^3 y z + 90240 x^3 z^2 + 397440 x^2 y^3 \\
& + 915840 x^2 y^2 z + 701280 x^2 y z^2 + 178400 x^2 z^3 + 440640 x y^4 + 1373760 x y^3 z \\
& + 1602720 x y^2 z^2 + 829200 x y z^3 + 160500 x z^4 + 182736 y^5 + 719280 y^4 z + 1130760 y^3 z^2 \\
& + 887400 y^2 z^3 + 347625 y z^4 + 54375 z^5 + 2560 x^4 + 36480 x^3 y + 26880 x^3 z + 144720 x^2 y^2 \\
& + 220320 x^2 y z + 83520 x^2 z^2 + 222480 x y^3 + 517320 x y^2 z + 399960 x y z^2 + 102800 x z^3 \\
& + 118260 y^4 + 370980 y^3 z + 435645 y^2 z^2 + 226950 y z^3 + 44250 z^4 + 2240 x^3 + 19920 x^2 y \\
& + 14960 x^2 z + 48240 x y^2 + 74280 x y z + 28500 x z^2 + 35100 y^3 + 82260 y^2 z + 64125 y z^2 \\
& + 16625 z^3 + 800 x^2 + 4176 x y + 3184 x z + 4680 y^2 + 7284 y z + 2826 z^2 + 96 x + 216 y \\
& + 168 z) / ((4 x + 6 y + 5 z) (4 x + 6 y + 5 z + 1) (4 x + 6 y + 5 z + 2) (4 x + 6 y + 5 z \\
& + 3) (4 x + 6 y + 5 z + 4) (3 y + 2 z) (3 y + 2 z + 1))
\end{aligned} \tag{4}$$

$$\begin{aligned}
& \text{VerifyWZForm}([f, g, h], [x, y, z]); \\
& \text{true}
\end{aligned} \tag{5}$$

>  $S := \text{WZFormDecomp}([f, g, h], [x, y, z]);$

$$\begin{aligned}
 S := 0, & \left[ \left[ \frac{1}{4x + 6y + 5z} + \frac{1}{4x + 6y + 5z + 1} + \frac{1}{4x + 6y + 5z + 2} \right. \right. \\
 & + \frac{1}{4x + 6y + 5z + 3}, \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} + \frac{1}{4} \right)} + \frac{1}{4 \left( x + \frac{5}{4} + \frac{3y}{2} + \frac{5z}{4} \right)} \\
 & + \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} \right)} + \frac{1}{4 \left( x + 1 + \frac{3y}{2} + \frac{5z}{4} \right)} + \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} + \frac{3}{4} \right)} \\
 & + \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} + \frac{1}{2} \right)}, \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} \right)} + \frac{1}{4 \left( x + 1 + \frac{3y}{2} + \frac{5z}{4} \right)} \\
 & + \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} + \frac{1}{4} \right)} + \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} + \frac{3}{4} \right)} \\
 & \left. + \frac{1}{4 \left( x + \frac{3y}{2} + \frac{5z}{4} + \frac{1}{2} \right)} \right], \left[ 0, \frac{1}{3y + 2z} + \frac{1}{3y + 2z + 1} + \frac{1}{3y + 2z + 2}, \right. \\
 & \left. \frac{1}{3 \left( y + \frac{2z}{3} \right)} + \frac{1}{3 \left( y + \frac{2z}{3} + \frac{1}{3} \right)} \right] \right]
 \end{aligned} \tag{6}$$

>  $\text{TransUniWZForms}(S[2], [x, y, z], Z);$

$$\text{table} \left( \left[ [0, 3, 2] = \frac{1}{Z}, [4, 6, 5] = \frac{1}{Z} \right] \right) \tag{7}$$

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