Hypergeometric Series Solutions of Linear Operator Equations

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Let $K$ be a field and $L: K[x] \rightarrow K[x]$ be a linear operator acting on the ring of polynomials in $x$ over the field $K$. We provide a method to find a suitable basis $\{b_k(x)\}$ of $K[x]$ and a hypergeometric term $c_k$ such that $y(x) = \sum_{k=0}^{\infty} c_k b_k(x)$ is a formal series solution to the equation $L(y(x)) = 0$. This method is applied to construct hypergeometric representations of orthogonal polynomials from the differential/difference equations or recurrence relations they satisfied. Both the ordinary cases and the $q$-cases are considered.

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