Integro-Differential Algebras, Operators, and Polynomials

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Motivated by boundary problems for linear ordinary differential equations (LODEs), the notion of integro-differential algebra combines a differential algebra with an integral operator. As in a differential Rota-Baxter algebra, the integral should be a right inverse of the derivation but we additionally require a suitable version of integration by parts. This so-called differential Baxter axiom allows us to define an "evaluation" in any integro-differential algebra, which is also the starting point for treating initial and boundary conditions in an algebraic setting.

We first review basic properties and examples of integro-differential algebras. Then we discuss the construction and some algebraic and algorithmic aspects of the associated ring of integro-differential operators. Integro-differential operators provide in particular an algebraic structure for computing with boundary problems for LODEs as well as their solution operators (Green's operators). As a second basic algebraic structure, we also outline canonical forms for integro-differential polynomials generalizing the usual differential polynomials.

This talk is based on joint work with Markus Rosenkranz.